

Unit 6: Trigonometric Equations and Identities ~ Learning Guide

Name: _____

Instructions:

Using a pencil, complete the following questions as you work through the related lessons. Show ALL work as it is explained in the lessons. Do your best and ask your instructor if you don't understand any question!

Lesson One

Solving Trigonometric Equations Using a Graphing Calculator:

1. Determine the number of solutions to each of these equations over the interval $0 \leq x < 2\pi$.

a) $\sin 2x = 0.3$

b) $\cos 5x = -0.7$

c) $\sin 6x = 1.2$

d) $3 \cos 4x = 2$

e) $\sin 2x = 1$

f) $2 \cos 6x + 1 = 2$

2. Solve each of the following by graphing calculator for $0 \leq x < 2\pi$, rounded to two decimal places. For each question provide a sketch, in the box, of the functions you graphed.

a) $\sin x = 0.2$

b) $\cos x = -0.7$

c) $\tan x = -1.5$

d) $2 \sin x = 1.05$

e) $3 + \sin x = 5 + 4 \sin x$



f) $\cos 2x = -0.13$



g) $\sin \frac{1}{2}x = 0.62$



h) $\cos^2 x + 2 \cos x - 1 = 0$



3. Solve each of the following by graphing calculator for general solutions rounded to two decimal places. For each question provide a sketch, in the box, of the functions you graphed.

a) $\sin 3x = 0.72$



b) $\cos(x + \pi) = 2 \cos x$



c) $\sin 2x = \cos 2x$



d) $2\sin^2 x + \sin x - 1 = 0$



*Lesson Two**Solving Trigonometric Equations for Exact Values:*

1. Solve for the exact values of θ , without a calculator, for $0 \leq \theta < 2\pi$.

a) $2 \sin \theta = 1$

b) $2 \cos \theta - \sqrt{3} = 0$

c) $3 \tan \theta + 3 = 0$

d) $2 \cos^2 \theta - 1 = 0$

e) $4 \sin \theta + \sqrt{12} = 0$

f) $\sqrt{2} \sin \theta - 1 = 0$

g) $\tan \theta - \sqrt{3} = 0$

h) $\tan^2 \theta - 1 = 0$

i) $\sqrt{2}\cos^2 \theta - \cos \theta = 0$

j) $4 \sin 2\theta - 2 = 0$

k) $\sqrt{2}\cos\frac{1}{2}\theta + 1 = 0$

l) $\sin^2 \theta + \sin \theta - 2 = 0$

m) $4 \cos^2 \theta - 1 = 0$

n) $(\sin \theta + 1)(2 \cos \theta + \sqrt{3}) = 0$

$$\text{o) } (\tan \theta + 1)(\sqrt{2} \sin \theta + 1)(2 \cos \theta - 1) = 0$$

Lesson Three

Reciprocal and Pythagorean Identities:

1. Solve for exact values of θ , without a calculator, for $0 \leq \theta < 2\pi$.
 - a) $\sec \theta - 2 = 0$
 - b) $\sin^2 \theta + \cos \theta = 1$

c) $2 \sin \theta - 3 \csc \theta = 1$

d) $2 \cot \theta + \sqrt{12} = 0$

2. Prove each of the following.

a) $\sin x \cot x = \cos x$

b) $\sec x \tan x = \frac{\sin x}{\cos^2 x}$

c) $\csc x = \frac{\sin^2 x + \cos^2 x}{\sin x}$

d) $1 + \sec x = \frac{1 + \cos x}{\cos x}$

e) $\cot x + \tan x = \sec x \csc x$

f) $1 - \sin x \cos x \cot x = \sin^2 x$

g) $1 - 2\cos^2 x = 2\sin^2 x - 1$

h) $1 - 2\sin^2 x = \cos^4 x - \sin^4 x$

*Lesson Four**More Trigonometric Identities:*

1. Prove each of the following.

a) $-\sec x = \frac{1 - \sec x}{1 - \cos x}$

b) $\frac{1}{\sec x} = \frac{\cos x \csc x}{\sin x + \cos x \cot x}$

$$\text{c) } -2\cot x = \frac{\sin x}{\cos x + 1} + \frac{\sin x}{\cos x - 1}$$

$$\text{d) } \csc x = \frac{1 + \sec x}{\sin x + \tan x}$$

$$\text{e) } \frac{\sec x}{1 - \sin x} = \frac{1 + \sin x}{\cos^3 x}$$

$$\text{f) } \csc x - \frac{\sin x}{1 + \cos x} = \frac{1}{\tan x}$$

$$\text{g) } \frac{\sec x + 1}{\sin^2 x} = \frac{\sec x}{1 - \cos x}$$

Lesson Five

Sum and Difference Identities:

1. Determine the exact value of the following trigonometric ratios using a sum or difference identity.

a) $\cos\left(\frac{5\pi}{12}\right)$

b) $\tan\left(-\frac{7\pi}{12}\right)$

2. Simplify the following by using a sum or difference identity.

a) $\sin 3x \cos 5x + \sin 5x \cos 3x$

b) $\frac{\tan 7x - \tan 2y}{1 + \tan 7x \tan 2y}$

3. Simplify each of the following using a sum or difference identity.

a) $\sin(x - \pi)$

b) $\cos\left(\frac{\pi}{2} - x\right)$

c) $\sin\left(x - \frac{\pi}{4}\right)$

d) $\cos\left(x + \frac{4\pi}{3}\right)$

- [illegible]

Lesson Six

Double Angle Identities:

1. Expand each double angle identity.

a) $5 \sin 6x$

b) $\frac{1}{2} \sin \frac{a}{2} x$

c) $6 \tan 4x$

d) $\frac{1}{8} \tan \frac{1}{2} x$

2. Write each as a single trigonometric identity.

a) $4 \sin 5x \cos 5x$

b) $3 \cos^2 2x - 3 \sin^2 2x$

c) $2 - 4 \sin^2 \frac{1}{2} x$

d) $\frac{1}{2} - \cos^2 3x$

e) $\sin^2 kx - \cos^2 kx$

f) $\frac{6 \tan 7x}{1 - \tan^2 7x}$

3. Solve algebraically for the exact value of θ given that $0 \leq \theta < 2\pi$.

a) $\cos \theta = -\sin 2\theta$

b) $\cos 2\theta = \sin \theta$

4. If $\cos \theta = \frac{2}{5}$ and θ is in quadrant IV, then determine the exact value of
a) $\sin 2\theta$
b) $\tan 2\theta$

5. If $\sin \theta = a$ and θ is in quadrant II, then determine the exact value of
 a) $\sin 2\theta$
 b) $\cos 2\theta$

6. Determine non-permissible values for θ given that $0 \leq \theta < 2\pi$.
- a) $\frac{\sec x}{1 - \sin x}$
- b) $\frac{\csc x}{\cos x - \sin x}$

7. Prove each of the following.

a) $-\csc^2 x = \frac{2}{\cos 2x - 1}$

b) $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

c) $\sin 2x - 2 \cot x \sin^2 x = 0$

d) $\frac{2 \cos x}{\sec^2 x} = \frac{1 + \cos 2x}{\sec x}$

e) $-\tan 2x = \frac{2}{\tan x - \cot x}$

f) $\sin 2x + 1 = (\sin x + \cos x)^2$

$$\text{g) } \cos x = \frac{2 \cos^2 x \csc x}{2 \cos^2 x \cot x + \sin 2x}$$

ANSWERSLesson One Solving Trigonometric Equations for Inexact Values

1. a) 4 b) 10 c) 0 d) 8 e) 2 f) 12 2. a) $x = 0.20, 2.94$ b) $x = 2.35, 3.94$
 c) $x = 2.16, 5.30$ d) $x = 0.55, 2.59$ e) $x = 3.87, 5.55$ f) $x = 0.85, 2.29, 3.99, 5.43$
 g) $x = 1.34, 4.95$ h) $x = 1.14, 5.14$
 3. a) $x = 0.27, 0.78 + \frac{2\pi}{3}n$ b) $x = 1.57, 4.71 + 2\pi n$ c) $x = 0.39, 1.96 + \pi n$ d) $x = 0.52, 2.62, 4.71 + \pi n$

Lesson Two Solving Trigonometric Equations for Exact Values

1. a) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ b) $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$ c) $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$ d) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ e) $\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$ f) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$
 g) $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$ h) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ i) $\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{4}$ j) $\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$
 k) $\theta = \frac{3\pi}{2}$ l) $\theta = \frac{\pi}{2}$ m) $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ n) $\theta = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}$ o) $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{3}$

Lesson Three Reciprocal and Pythagorean Identities

1. a) $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ b) $\theta = 0, \frac{\pi}{2}, \frac{3\pi}{2}$ c) $\theta = \frac{3\pi}{2}$ d) $\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$
 2. Answers may vary.

Lesson Four More Trigonometric Identities

1. Answers may vary.

Lesson Five Sum and Difference Identities

1. a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ b) $2 + \sqrt{3}$ 2. a) $\sin 8x$ b) $\tan(7x - 2y)$ 3. a) $-\sin x$ b) $\sin x$
 c) $\frac{\sin x - \cos x}{\sqrt{2}}$ d) $\frac{\sqrt{3}\sin x - \cos x}{2}$ 4. $\sin(5x - 4)$ 5. a) $\frac{2-2\sqrt{10}}{9}$ b) $\frac{4\sqrt{2}+\sqrt{5}}{9}$
 6. a) $\frac{6+4\sqrt{21}}{25}$ b) $\frac{8-3\sqrt{21}}{25}$

Lesson Six Double Angle Identities

1. a) $10 \sin 3x \cos 3x$ b) $\sin \frac{a}{4} x \cos \frac{a}{4} x$ c) $\frac{12 \tan 2x}{1 - \tan^2 2x}$ d) $\frac{1}{4} \left(\frac{\tan^{\frac{1}{4}} x}{1 - \tan^{\frac{1}{4}} x} \right)$
 2. a) $2 \sin 10x$ b) $3 \cos 4x$ c) $2 \cos x$ d) $-\frac{1}{2} \cos 6x$ e) $-\cos 2kx$ f) $3 \tan 14x$
 3. a) $\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ b) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ 4. a) $-\frac{4\sqrt{21}}{25}$ b) $\frac{4\sqrt{21}}{17}$
 5. a) $-2a\sqrt{1-a^2}$ b) $1 - 2a^2$ 6. a) $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$ b) $x = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$
 7. Answers may vary.